Optimizing an Investment

November 23, 2024

1 Premise

A person wants to invest in gold and silver and wants to maximize their returns for a given initial investment in a single year. However, they're not sure how much gold and silver to buy. Should they buy a lot of gold and less silver? The other way around? Equal amounts?

2 Assumptions

1. Assume that the initial investment is $100,000.

2. Assume that the annual return rates for silver and gold are 3.96% and 6.57% respectively as calculated previously from the dataset.

3. Because silver is significantly cheaper than gold, the person wants to invest in at least one and a half times as much silver than gold.

4. The person do not want to invest more than $75000 into a single commodity.

3 Model Formulation

Let g represent the amount of money invested into gold.

Let s represent the amount of money invested into silver.

From 1, we get $g+s \leq 100000$ i.e. the total spent must not exceed the initial investment.

From 2, we create a return function: $0.0657g + 0.0396s$ using the interest rates calculated.

From 3, we get $1.5s \geq g$ as we want at least four times as much silver as gold.

From 4, we get $s \leq 75000$ and $g \leq 75000$ as the person does not want to put more than the allocated amount into a single metal.

Also, we get $g \geq 0$ and $s \geq 0$, as the person can theoretically invest into only one metal and not the other and investment capital cannot be negative.

Using these assumptions, we can create a linear programming problem:

Maximize $0.0657g + 0.0396s$ subject to the constraints

$g + s \leq 100000$

$g - 1.5s \leq 0$

$g \leq 75000$

$s \leq 75000$

$g, s \geq 0$

4 Problem Computation

First, we convert this standard linear programming problem into a canonical form problem by introducing slack variables to every constraint to get equalities. Using some real numbers a, b, c, d:

$g + s + a = 100000$

$g - 1.5s + b = 0$

$g + c = 75000$

$s + d = 75000$

We will now convert this into a matrix:

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & -1.5 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} g \\ s \\ a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 100000 \\ 0 \\ 75000 \\ 75000 \end{pmatrix}$$

The rank (number of linearly independent rows) is 4 and the output vector has all positive entries, therefore we can use the simplex method to solve this problem.

Let T_n represent the n^{th} tableau

T_1/ Initial Tableau=

-	g	s	a	b	c	d	-
a	1	1	1	0	0	0	100000
b	1	-1.5	0	1	0	0	0
c	1	0	0	0	1	0	750000
d	0	1	0	0	0	1	750000
obj	-0.0657	-0.0396	0	0	0	0	0

We choose the column with the most negative entry, which is the g column. We then consider the lowest ratio among all of the last column to each entry of the g column, which is $\frac{0}{1} = 0$. We now want 1 in this entry and zero elsewhere in the column. We already have 1 in the entry, so now we do get zeroes in the other parts of the g column:

T_2=

-	g	s	a	b	c	d	-
a	0	2.5	1	-1	0	0	100000
g	1	-1.5	0	1	0	0	0
c	0	1.5	0	-1	1	0	750000
d	0	1	0	0	0	1	750000
obj	0	-0.13815	0	0.0657	0	0	0

We continues as we need all the values in the last row to be positive. We choose the column with the most negative entry, which is the s column. We then consider the lowest ratio among all of the last column to each entry of the s column, which is $\frac{100000}{2.5} = 40000$. We now want 1 in this entry and zero elsewhere in the column.

T_3=

-	g	s	a	b	c	d	-
s	0	1	0.4	-0.4	0	0	40000
g	1	0	0.6	0.4	0	0	60000
c	0	0	-0.6	-0.4	1	0	150000
d	0	0	-0.4	0.4	0	1	350000
obj	0	0	0.05526	0.01044	0	0	5526

The simplex terminates as there are no negative entries in the last row.

5 Solution

Based on the algorithm used, the ideal amount to invest in silver is $40000 and the amount to invest in gold is $60000 with a maximum annual return of $5526.

Using the prices of $23.87 for silver and $2068.10 for gold, this corresponds to approximately 1675.74 troy ounces of silver and 29.01 troy ounces of gold.