Future and Present Values

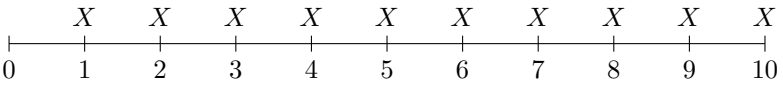
December 11, 2024

1 Future Values

Suppose we want to know how much silver and gold we need to buy per year to accumulate to $100000 a decade from now using the current average yearly return rates of 3.96% and 6.57% as before.

1.0.1 Silver

Let X denote the yearly amount of silver purchased per year and assume that the yearly effective rate of return is 3.96%. We can construct a timeline for these payments:



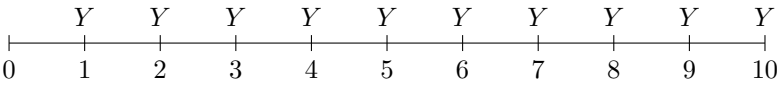
This forms a type of cash flow called an annuity immediate, with payments being made at the end of each period.

Then $Xs_{\overline{10}|3.96\%} = 100000$, where $s_{\overline{n}|i} = \frac{(1+i)^n - 1}{i}$ is the formula to accumulate payments, n is the amount of payment periods and i is the effective interest rate per period, in this case our rate of return per year. We have 10 years and a rate of return of 3.96%.

So $X\frac{(1+0.0396)^{10}-1}{0.0396} = 100000$, therefore $X = \$8344.56$ i.e. you need to buy $8344.56 worth of silver every year to accumulate $100000 a decade later.

1.0.2 Gold

Let Y denote the yearly amount of silver purchased per year and assume that the yearly effective rate of return is 6.57%. We can construct a timeline for these payments:



This forms another annuity immediate, with payments being made at the end of each period.

Then $Y s_{\overline{10}|6.57\%} = 100000$, where $s_{\overline{n}|i} = \frac{(1+i)^n - 1}{i}$ is the formula to accumulate payments, n is the amount of payment periods and i is the effective interest rate per period, in this case our rate of return per year. We have 10 years and a rate of return of 6.57%.

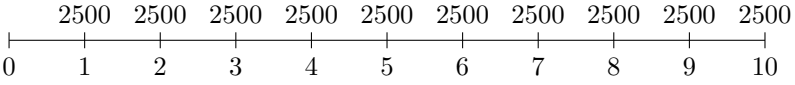
So $Y \frac{(1+0.0657)^{10} - 1}{0.0657} = 100000$, therefore $Y = \$7386.07$ i.e. you need to buy \$7386.07 worth of gold every year to accumulate \$100000 a decade later.

2 Present Values

Now suppose you purchased silver and gold that pays consistent dividends of $2500 per year for 10 years. How much are these payments worth today?

2.0.1 Silver

Let A denote the present value of the dividends and assume that the yearly effective rate of return is 3.96%. We can construct a timeline for these payments:



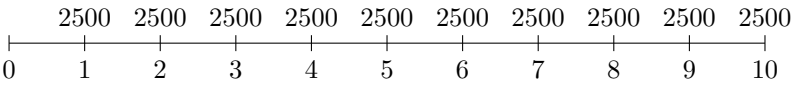
This forms an annuity immediate, with dividends being paid at the end of each period.

Then $A = 2500a_{\overline{10}|3.96\%}$, where $a_{\overline{n}|i} = \frac{1-\nu^n}{i}$ is the formula to discount payments using a discount factor of ν, which is equal to $(1+i)^{-1}$, n is the amount of payment periods and i is the effective interest rate per period, in this case our rate of return per year. We have 10 years and a rate of return of 3.96%.

So $A = 2500\frac{1-(1.0396)^{-10}}{0.0396} = 20317.68$ i.e. the present value of these dividends is 20317.68

2.1 Gold

Let B denote the present value of the dividends and assume that the yearly effective rate of return is 6.57%. We can construct a timeline for these payments:



This forms an annuity immediate, with dividends being paid at the end of each period.

Then $B = 2500a_{\overline{10}|6.57\%}$, where $a_{\overline{n}|i} = \frac{1-\nu^n}{i}$ is the formula to discount payments using a discount factor of ν, which is equal to $(1+i)^{-1}$, n is the amount of payment periods and i is the effective interest rate per period, in this case our rate of return per year. We have 10 years and a rate of return of 6.57%.

So $A = 2500\frac{1-(1.0657)^{-10}}{0.0657} = 17913.35$ i.e. the present value of the dividends is 17913.35 .